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How the exponent of even element of a N-equation exceeds two.

$a^x + b^y = c^z$, where there is no common factor among a, b, c.

if b is the even element then the nature of N-equation where b is in power form is $a^2 + b^{2n} = c^2$, where $n \in I^+$

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ABSTRACT

Regarding Beal equation mystery all the important sides have been discussed in Aug-edition and in subsequent development of Oct-edition 2013. It has been shown that how left hand odd element of a N-equation produces its power beyond two and then other two exponents are bound to be restricted on two. Similar occurrence is noticed when right hand odd element produces its power beyond two. Obviously, when the even element produces power beyond two the exponents of both odd elements are bound to be restricted on two. But how the even element produces its power beyond two was not elaborately discussed in earlier two editions. This paper contains a little proof or method by which we can understand the power characteristics of even element of a N-equation.

Keywords

Mixed Zygote form & Odd zygote form, N-equation, N_d operation & N_s operation, NZ-equation,

I. Introduction

Let us consider a N-eq. $a^2 + b^2 = c^2$ where a & c are odd and b is even elements. We have found that a & c both are capable of increasing its power (not simultaneously) even & odd both. why? Because a^2 & a both can be expressed in the form of $\alpha^2 - \beta^2$. So N_d operation can be applied between a^2 & a or a^2 & a^2 or a & a. Similarly, c^2 & c both are expressible in the form of $\alpha^2 + \beta^2$. So N_s operation can be applied between c^2 & c or c^2 & c^2 or c & c. But what happens when b produces power beyond 2?

II. How the exponent of even element of a N-equation exceeds two.

Earlier we discussed the Zygote form of N-equation. Let us now introduce another form i.e. 'Odd zygote form' of N-equation & the previous one may be renamed as 'Mixed Zygote form'.
We have $c + b = d_1^2$ & $c - b = d_2^2$. $\therefore$ $c = (d_1^2 + d_2^2)/2$ & $b = (d_1^2 - d_2^2)/2$
& $a = \sqrt{(c^2 - b^2)} = \sqrt{\{(c + b)(c - b)\}} = d_1d_2$.

$\Rightarrow \{(d_1^2 - d_2^2)/2\}^2 + (d_1d_2)^2 = \{(d_1^2 + d_2^2)/2\}^2$ Here, d_1 & d_2 both are odd.

The example chart can be given below.

1st kind

For k = 1,
$(1.3)^2 + \{(3^2 - 1^2)/2\}^2 = \{(3^2 + 1^2)/2\}^2$
$(1.5)^2 + \{(5^2 - 1^2)/2\}^2 = \{(5^2 + 1^2)/2\}^2$
$(1.7)^2 + \{(7^2 - 1^2)/2\}^2 = \{(7^2 + 1^2)/2\}^2$
$(1.9)^2 + \{(9^2 - 1^2)/2\}^2 = \{(9^2 + 1^2)/2\}^2$
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For k = 9,
$(3.9)^2 + \{(9^2 - 3^2)/2\}^2 = \{(9^2 + 3^2)/2\}^2$
$(3.11)^2 + \{(11^2 - 3^2)/2\}^2 = \{(11^2 + 3^2)/2\}^2$
$(3.13)^2 + \{(13^2 - 3^2)/2\}^2 = \{(13^2 + 3^2)/2\}^2$
$(3.15)^2 + \{(15^2 - 3^2)/2\}^2 = \{(15^2 + 3^2)/2\}^2$
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For k = 25,
$(5.13)^2 + \{(13^2 - 5^2)/2\}^2 = \{(13^2 + 5^2)/2\}^2$
$(5.15)^2 + \{(15^2 - 5^2)/2\}^2 = \{(15^2 + 5^2)/2\}^2$
$(5.17)^2 + \{(17^2 - 5^2)/2\}^2 = \{(17^2 + 5^2)/2\}^2$
$(5.19)^2 + \{(19^2 - 5^2)/2\}^2 = \{(19^2 + 5^2)/2\}^2$
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2^{nd} kind.

For k = 2,
$\{(5^2 - 3^2)/2\}^2 + (5.3)^2 = \{(5^2 + 3^2)/2\}^2$
$\{(7^2 - 5^2)/2\}^2 + (7.5)^2 = \{(7^2 + 5^2)/2\}^2$
$\{(9^2 - 7^2)/2\}^2 + (9.7)^2 = \{(9^2 + 7^2)/2\}^2$
$\{(11^2 - 9^2)/2\}^2 + (11.9)^2 = \{(11^2 + 9^2)/2\}^2$
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For k = 8,
$\{(7^2 - 3^2)/2\}^2 + (7.3)^2 = \{(7^2 + 3^2)/2\}^2$
$\{(9^2 - 5^2)/2\}^2 + (9.5)^2 = \{(9^2 + 5^2)/2\}^2$
$\{(11^2 - 7^2)/2\}^2 + (11.7)^2 = \{(11^2 + 7^2)/2\}^2$
$\{(13^2 - 9^2)/2\}^2 + (13.9)^2 = \{(13^2 + 9^2)/2\}^2$
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For k = 18,
$\{(11^2 - 5^2)/2\}^2 + (11.5)^2 = \{(11^2 + 5^2)/2\}^2$
$\{(13^2 - 7^2)/2\}^2 + (13.7)^2 = \{(13^2 + 7^2)/2\}^2$
$\{(15^2 - 9^2)/2\}^2 + (15.9)^2 = \{(15^2 + 9^2)/2\}^2$
$\{(17^2 - 11^2)/2\}^2 + (17.11)^2 = \{(17^2 + 11^2)/2\}^2$

Here, b^2 is expressible in the form of $\alpha_1^2 - \beta_1^2$ whereas b is expressible in the form of $(\alpha_1^2 - \beta_1^2)/2$ where α, β both are odd. So by N_d operation we cannot receive a relation like $b^3 = \alpha^2 - \beta^2$ But for b^4 by N_d operation we can always get a relation $(b.b/2)^2 = p^2 - q^2$ where p, q both are odd.
Hence, (any even no.)$^{\text{any odd no.}}$ cannot be a term of N-equation. Also, 2(any odd no.) cannot be the element of N-equation. They are under NZ-equation.

So it is observed that 'a' produces power by N_d operation among mixed zygote expressions i.e. mixed with odd & even elements.
'b' produces power by N_d operation among odd zygote expressions i.e. mixed with only odd elements.
'c' produces power by N_s operation among mixed zygote expressions.
Any two of these three operations or all the three are not possible to run simultaneously.
So in N-equation only one element can raise its power beyond two.

The general form of N-equation where even element (b) is in power form by continuous applications of N_d operations over b^2 & b^2,can be written as

$(b^n)^2 + ({}^nc_1c^{n-1}a + {}^nc_3c^{n-3}a^3 + \ldots\ldots)^2 = (c^n + {}^nc_2c^{n-2}a^2 + \ldots..)^2$
Which is a composite set with common factor 2^{n-1}.

$\Rightarrow (b^n/2^{n-1})^2 + (d_1)^2 = (d_2)^2$, where obviously, d_1 & d_2 are odd.
Say, $b = 2^m.\alpha^p$ where α is odd.
$\Rightarrow \{2^{n(m-1)+1}.\alpha^{pn}\}^2 + (d_1)^2 = (d_2)^2$, where GCF of n(m – 1) + 1 & pn > 1so as to receive the even element in power form.

III. Conclusion

With the help of N-equation it has been possible to analyze all the important aspects of Beal-equation. It also covers the proof of Fermat's Last Theorem i.e. $a^n + b^n = c^n$ where n is positive integer > 2, does not have any solution. Now we can put our attention over the fact $a^n + b^n = c^n + d^n$. For n = 2 all the relations are made available from the right hand element (c) of N-equation. Any composite number of c having at least two prime factors can produce such type of relations. But what happens when n > 2? It will be worthy to mention here that for n = 3, the minimum number having the property $a^3 + b^3 = c^3 + d^3$, was first noticed by great mathematician Sir Srinivasa Ramanujan i.e. $1^3 + 12^3 = 9^3 + 10^3 = 1729$ (Ramanujan number). But is there any relations among a, b, c, d or can all the relations be arranged in a systematic manner like N-eq?

References

Books

[1] Academic text books of Algebra.

[2] IJSER Vol-4, Aug-edition & Oct-edition, Author: DEBAJIT DAS.